MATH5360 Game Theory
Exercise 3

Assignment 3: 1(a)(b), 2(b)(d), 4(a)(b), 5, 9(a)(c) (Due: 23 March 2020 (Monday))

1. Find all Nash equilibria of the following bimatrix games. For each of the Nash equilibrium, find the payoff pair.

   (a) $\begin{pmatrix} (1,4) & (5,1) \\ (4,2) & (3,3) \end{pmatrix}$

   (b) $\begin{pmatrix} (5,2) & (2,0) \\ (1,1) & (3,4) \end{pmatrix}$

   (c) $\begin{pmatrix} (1,5) & (2,3) \\ (5,2) & (4,2) \end{pmatrix}$

   (d) $\begin{pmatrix} (-1,0) & (2,1) \\ (4,3) & (-3,-1) \end{pmatrix}$

2. Find all Nash equilibria of the following bimatirx games

   (a) $\begin{pmatrix} (4,1) & (2,3) & (3,4) \\ (3,2) & (5,5) & (1,2) \end{pmatrix}$

   (b) $\begin{pmatrix} (1,0) & (4,-1) & (5,1) \\ (3,2) & (1,1) & (2,-1) \end{pmatrix}$

   (c) $\begin{pmatrix} (4,6) & (0,3) & (2,-1) \\ (2,4) & (6,5) & (-1,1) \\ (5,0) & (1,2) & (4,3) \end{pmatrix}$

   (d) $\begin{pmatrix} (3,2) & (4,0) & (7,9) \\ (2,6) & (8,4) & (3,5) \\ (5,4) & (5,3) & (4,1) \end{pmatrix}$

3. The Brouwer's fixed-point theorem states that every continuous map $f : X \to X$ has a fixed-point if X is homeomorphic to a closed unit ball. Find a map $f : X \to X$ which does not have any fixed-point for each of the following topological spaces X. (It follows that the following spaces are not homeomorphic to a closed unit ball.)

   (a) X is the punched closed unit disc $D^2 \setminus \{0\} = \{(x,y) \in \mathbb{R}^2 : 0 < x^2 + y^2 \leq 1\}$

   (b) X is the unit sphere $S^2 = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$

   (c) X is the open unit disc $B^2 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

4. For each of the following bimatrices (A, B), find the values ν_A and ν_{B^T} of A and B^T respectively, and the Nash bargaining solution using $(\mu, \nu) = (\nu_A, \nu_{B^T})$ as the status quo point.

   (a) $\begin{pmatrix} (4,-4) & (-1,-1) \\ (0,1) & (1,0) \end{pmatrix}$

   (b) $\begin{pmatrix} (3,1) & (1,0) \\ (0,-1) & (2,3) \end{pmatrix}$

   (c) $\begin{pmatrix} (2,2) & (0,1) & (1,-1) \\ (4,1) & (-2,1) & (1,3) \end{pmatrix}$

   (d) $\begin{pmatrix} (6,4) & (0,10) & (4,1) \\ (8,-2) & (4,1) & (0,1) \end{pmatrix}$

5. Two broadcasting companies, NTV and CTV, bid for the exclusive broadcasting rights of an annual sports event. If both companies bid, NTV will win the bidding with a profit of \$20 (million) and CTV will have no profit. If only NTV bids, therell be a profit of \$50 (million). If only CTV bids, therell be a profit of \$40 (million). Find the Nash's solution to the bargaining problem.

6. Let $\mathcal{R} = \{(u,v) : v \geq 0 \text{ and } u^2 + v \leq 4\} \subset \mathbb{R}^2$. Find the arbitration pair $A(\mathcal{R}, (\mu, \nu))$ using the following points as the status quo point (μ, ν).

(a) $(0,0)$ (b) $(0,1)$

7. Let $\mathcal{R} \subset \mathbb{R}^2$ be a closed and bounded convex set, $(\mu,\nu) \in \mathcal{R}$ and $(\alpha,\beta) = A(\mathcal{R},(\mu,\nu))$ be the arbitration pair with $\alpha \neq \mu$. Suppose the boundary of $\mathcal{R}$ is given, locally at (α,β), by the graph of a differentiable function $f(x)$ with $f(\alpha)=\beta$. Prove that $f'(\alpha)$ is equal to the negative of the slope of the line joining (μ,ν) and (α,β).

8. Suppose A is an $n \times n$ matrix such that the sum of entries in any row of A is equal to a constant rn. Let (μ,ν) be the status quo point of the bimatrix (A, A^T).

   (a) Prove that there is a Nash equilibrium of (A,A^T) with (r,r) as payoff pair.

   (b) Prove that the arbitration payoff pair of the bimatrix (A,A^T) is $(\alpha,\beta)=(m,m)$ where m is the maximum entry of $\dfrac{A+A^T}{2}$. (Here in finding the arbitration payoff pair of bimatix (A,B), the status quo point is taken to be $(\mu,\nu) = (v(A), v(B^T))$ where v is the value of a matrix.)

9. Find the threat strategies and the threat solutions of the following game bimatrix.

   (a) $\begin{pmatrix} (3,-2) & (2,4) \\ (1,0) & (3,-1) \end{pmatrix}$

   (b) $\begin{pmatrix} (5,3) & (1,3) \\ (4,4) & (2,1) \end{pmatrix}$

   (c) $\begin{pmatrix} (6,4) & (2,3) & (4,7) \\ (2,6) & (4,2) & (5,4) \end{pmatrix}$

   (d) $\begin{pmatrix} (2,8) & (7,5) & (6,3) \\ (0,7) & (4,3) & (5,5) \\ (3,-1) & (-2,6) & (2,7) \end{pmatrix}$